

## Errata of Modern Nonconvex Nondifferentiable Optimization

July 29, 2022

- Page 5, the first paragraph of the proof of Proposition 1.1.1.

Replace the sentence “For each positive integer  $\nu$ , let  $i_\nu$  be the smallest integer greater than  $\nu$  such that  $\|x^{i_\nu} - x^\infty\| > \delta$ ” with the following one:

Let  $\{x^{j_\nu}\}$  with  $j_{\nu+1} > j_\nu$  be a subsequence in the neighborhood of  $\mathcal{N}(x^\infty, \delta)$  converging to  $x^\infty$ . For each  $\nu$ , let  $i_\nu$  be the smallest integer  $i > \max(j_\nu, i_{\nu-1})$  such that  $\|x^{i_\nu} - x^\infty\| > \delta$ .

- Page 5, the last line. The second  $S_{\nu_1}$  should be  $S_{\nu_2}$ .
- Page 19, the last line. The term  $x = x^* + J_x F(x^*, y^*)^{-1}[e(x, y) + F(x^*, y)]$  should be  $x = x^* - J_x F(x^*, y^*)^{-1}[e(x, y) + F(x^*, y)]$ .
- Page 20, the third display,  $x(y) - x(z) = \dots$  should be  $x(z) - x(y) = \dots$ .
- Page 20, the display below (1.15) and the last display on this page, the most right sides should have an additional term  $o(\|x(y^* + h) - x(y^*)\|)$ .
- Page 26, Exercise 1.2.4. Change to the following:

For a vector  $x$  (or a matrix  $A$ ), let  $|x|$  (or  $|A|$ ) be the vector (or the matrix) whose components are equal to the absolute values of the corresponding components of  $x$  (or  $A$ ). A norm  $\|\bullet\|$  on  $\mathbb{R}^n$  is said to be *monotonic* if  $|x| \leq |y|$  implies  $\|x\| \leq \|y\|$  for all  $x$  and  $y$  in  $\mathbb{R}^n$ . Similarly, a norm  $\|\|\bullet\|\|$  on  $\mathbb{R}^{n \times n}$  is monotonic if  $|A| \leq |B|$  implies  $\|\|A\|\| \leq \|\|B\|\|$ .

Show the following:

- (a) A vector norm  $\|\bullet\|$  is monotonic if and only if  $\|x\| = \||x|\|$  for all  $x \in \mathbb{R}^n$ .
- (b) A matrix norm induced by a monotonic vector norm is monotonic.

- Page 29, the last inequality in the proof of Proposition 1.2.3,

$$\|\|(\mathbb{I} - A)^{-1}\|\| \geq \frac{\|\|\mathbb{I}\|\|}{\|\|\mathbb{I} + A\|\|}$$

should be

$$\|\|(\mathbb{I} - A)^{-1}\|\| \geq \frac{\|\|\mathbb{I}\|\|}{\|\|\mathbb{I} - A\|\|}.$$

- Page 32, line 13. “each column of  $Q$  is an eigenvector of the corresponding diagonal entry of  $A$ ” should be “...of the corresponding diagonal entry of  $D$ ”.
- Page 39, the proof from (h)  $\Rightarrow$  (i) only shows the nonnegativity of  $\text{trace}(AB)$  and does not show this term is strict positive if  $B \neq 0$ .
- Page 43, line 18. A “ $\beta$ ” is missing in the third equation. The correct one should be

$$Ar = Aq - \beta Ap = e^{i\theta_2} Az - \beta e^{i\theta_1} Ax = \rho(A)r.$$

- Page 51, line -2. “ $n$ ” should be  $\ell$ .
- Page 53, line -3 in proof of Proposition 1.4.1. The exponent  $-1$  in  $\beta_{\nu-1}$  should be removed.
- Page 56, fifth line below Exercise 1.5.1,  $\Phi : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$  should be  $\Phi : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$ .
- Page 58, Theorem 1.5.1. Let  $\Phi : \mathcal{O} \rightrightarrows \mathbb{R}^m$  should be  $\Phi : \mathcal{O} \rightrightarrows \mathbb{R}^n$ .

- Page 66, Proposition 2.1.1(3). It should be: there exists  $y \geq 0$  such that  $A^\top y < 0$ .
- Page 91, Lemma 2.5.1. The correct statements should be:  
Suppose that  $f$  is continuously differentiable. Let  $\{x^\nu\}$  be an infinite sequence generated by Algorithm 2.5.1. If  $\{x^\nu\}$  and  $\{d^\nu\}$  are bounded, then the following statements must hold:  
(a) The sequence  $\{f(x^\nu)\}$  converges.  
(b)  $\lim_{\nu \rightarrow \infty} \nabla f(x^\nu)^\top d^\nu = 0$ .
- Page 92, Theorem 2.5.1. Need to assume that the direction  $d^\nu$  to be *gradient related* to  $\{x^\nu\}$ , i.e., for any subsequence of  $\{x^\nu\}_{\nu \in \kappa}$  converging to a nonstationary point,  $\limsup_{\nu \rightarrow \infty} \nabla f(x^\nu)^\top d^\nu < 0$ .
- Page 134. The term  $Y^\top Z$  should be  $\text{tr}(Y^\top Z)$ .
- Page 140. The closure of the set is  $\mathbb{R}^2$ .
- Page 167, line 14. The term  $x'$  should be  $w'$ .
- Page 475, Lemma 8.3.1, the convexity of the set  $X$  needs to be dropped (the proof remains valid) because in the application to Proposition 8.5.6, the set  $S_c(x)$  may not be convex.
- Page 536, Theorem 9.2.1 and related results, the Lipschitz continuity of  $f$  on the closed set  $X$  should be replaced by its Lipschitz continuity on an open set where the boundedness condition:  $f'(\bar{x}; v) \leq L\|v\|$  holds; e.g., in a neighborhood of  $\bar{x}$ .
- Page 663, line 2. The term  $p^q$  should be  $p^{-q}$ .
- Page 667, one line before expression (11.9): “be” should be “being”; first line after expression (11.9):  $b_{i1}$  should be  $\beta_{i1}$ .
- Page 671, second bullet in statement of Proposition 11.2.1,  $\nabla_x$  should be  $\nabla_{x^i}$ .
- Page 672, Proposition 11.2.2 needs the map  $\Phi$  to be nonempty-valued and compact-valued.
- Page 678, Proposition 11.2.3 requires the same nonempty-valuedness and compact-valuedness of  $\Phi$ .
- Page 679, line -3 before end of proof:  $\tau$  should be  $\tau_\nu$ .
- Page 679, Exercise 11.3.1. First line  $\{\widehat{f}\}_{i=1}^N$  should be  $\{\widehat{f}_i\}_{i=1}^N$ .
- Page 687, 5 lines below Example 11.4.1: asterisk \* is missing in  $x^{-i}$ .
- Page 696, expression (11.36):  $-g(x)$  should be on the next line.
- Page 696, line -4: “constraint”  $-j$  “condition”.
- Page 699, 2 lines in proof of Lemma 11.5.1:  $\widehat{\Xi}$  should be  $\overline{\Xi}$  and a superscript transpose  $^\top$  is missing after  $\bar{\pi}$ .
- A lot of places: *implicit* convex-concave functions should be *implicitly* convex-concave functions.